Fall 2006, CS 598CC: Approximation Algorithms
Homework 2
Due: 10/31/2006 in class

Instructions and Policy: You are not allowed to consult any material outside of the textbook and class notes in solving these problems. Unless otherwise noted you are to solve the problems on your own. Even when collaboration is allowed, each person should write up their own solutions independently. You need to indicate the names of the people you discussed a problem with. Ideally you should discuss with no more than two other people.

Please write clearly and concisely - clarity and brevity will be rewarded. Refer to known facts as necessary. Your job is to convince me that you know the solution, as quickly as possible.

For this homework collaboration is allowed on all problems.

Problem 1 [10 pts] The feasibility problem for linear programming is the following. Given a set of inequalities that define a polyhedron P, is P empty? The input size is the sum of the sizes of the binary representations of all the numbers in the inequalities. The optimization problem for linear programming consists of a set of inequalities that define a polyhedron P and a vector c and the goal is to find $\min cx, x \in P$. Show how you can obtain a polynomial time algorithm for the optimization problem assuming a polynomial time algorithm for the feasibility problem. (Hint: you can use binary search and/or duality but be formal in your arguments about polynomial time).

Problem 2 [20 pts] Exercise 13.4 from Vazirani's book. You can use any method to prove the approximation bounds.

Problem 3 [20 pts] Consider the profit maximization version of the generalized assignment problem. You are given n items and m bins. The size of items j in bin i is s_{ij} and the profit for packing item j in bin i is p_{ij}. Bin i has capacity b_i. The goal is to pack a maximum profit subset of items into the bins such that the items do not violate the bin capacity constraints. Note that by adding a dummy $m+1$th bin with $s_{(m+1)j} = p_{(m+1),j} = 0$ for all j, we can assume that all items are packed.

- Using the 2-approximation algorithm presented in class for the cost minimization version of generalized assignment, show how one can get a $1/2$ approximation for the profit maximization. You can accomplish this using the following steps. a) reduce profit maximization to cost minimization using the fact that all items are packed. b) use the property of the algorithm for cost minimization which guarantees optimum cost (and hence optimum profit for the original problem) but violates the bin capacity constraints in a very specific form. c) show how you can alter the solution for the cost minimization problem (by removing some items) to obtain a feasible solution that does not violate any constraints and still guarantees an approximation ratio of $1/2$.

- Obtain a PTAS for the problem when m is a fixed constant independent of n as follows. (Do you see that the problem does not admit an FPTAS even for $m = 2$?) Guess the most profitable m/ϵ items and their assignment to the bins. Then solve an LP for the remaining items. Use a basic feasible solution to the LP and assign only the items which are integrally assigned by the LP. Why does this work? (Hint: how many items do you discard from the basic feasible solution and what is their profit?). What is the running time of the algorithm? (Do you see that a similar approach works for the k-dimensional knapsack problem when k is a fixed constant independent of the input size?).